

# 非局部时滞竞争扩散系统行波解\*

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**摘要:** 研究了非局部时滞竞争系统行波解。将不动点定理与广义上下解结合证明了行波解的存在性, 利用压缩矩形的思想得到了行波解的渐近性态, 最后利用渐近传播理论研究了行波解的不存在。

**关键词:** 压缩矩形; 渐近传播; 入侵与共存

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## Traveling Wave Solutions of a Competition Diffusion System with Nonlocal Delays

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**Abstract:** The traveling wave solutions of a competitive system with nonlocal delay are concerned. By combining fixed point theorem with generalized upper and lower solutions, the existence of traveling wave solutions is established. Then the asymptotic behavior of traveling wave solutions is obtained by the idea of contracting rectangles. Finally, the nonexistence of traveling wave solutions is proved by the theory of asymptotic spreading.

**Key words:** contracting rectangle; asymptotic spreading; invasion and coexistence

本文研究如下具有非局部时滞竞争系统的行波

解<sup>[1-2]</sup>

$$\begin{cases} \frac{\partial u_1}{\partial t} = d_1 \frac{\partial^2 u_1}{\partial x^2} + r_1 u_1(x, t) [1 - a_1 u_1(x, t) - b_1 (g_1 * u_1)(x, t) - c_1 (g_2 * u_2)(x, t)], \\ \frac{\partial u_2}{\partial t} = d_2 \frac{\partial^2 u_2}{\partial x^2} + r_2 u_2(x, t) [1 - a_2 u_2(x, t) - b_2 (g_3 * u_2)(x, t) - c_2 (g_4 * u_1)(x, t)] \end{cases} \quad (1)$$

这里

$$(g_i * u_j)(x, t) = \int_{-\infty}^t \int_{\mathbf{R}} g_i(y, s) u_j(x - y, t - s) dy ds, \\ i \in \{1, 2, 3, 4\}, j \in \{1, 2\}$$

其中  $d_i, a_i, b_i, c_i$  是正常数,  $x \in \mathbf{R}, t > 0$ , 并且  $g_i, i = 1, 2, 3, 4$ , 是种群动力系统中描述个体在历史上随机游走的核函数<sup>[3-5]</sup>, 且满足

$$g_i(-x, t) = g_i(x, t),$$

$$\int_0^{+\infty} \int_{-\infty}^{+\infty} g_i(y, s) dy ds = 1, i = 1, 2, 3, 4 \quad (2)$$

显然, 方程 (1) 有一个平凡的平衡点  $E_0 = (0, 0)$ , 两个半平凡的稳定态

$$E_1 = \left( \frac{1}{a_1 + b_1}, 0 \right), \quad E_2 = \left( 0, \frac{1}{a_2 + b_2} \right)$$

如果  $a_2 + b_2 > c_1, a_1 + b_1 > c_2$  或  $a_2 + b_2 < c_1, a_1 + b_1 < c_2$ , 则方程 (1) 还有一个正的稳定态

$$E^* = \left( \frac{a_2 + b_2 - c_1}{(a_1 + b_1)(a_2 + b_2) - c_1 c_2}, \frac{a_1 + b_1 - c_2}{(a_1 + b_1)(a_2 + b_2) - c_1 c_2} \right) = (k_1, k_2)$$

在方程 (1) 中取一些特殊的核函数时, 其行波解已被广泛研究。特别地, 如果  $g_i(x, t) = \delta(x)\delta(t)$  (此处  $\delta(\cdot)$  是狄拉克函数), 此时系统 (1) 变成

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$$\begin{cases} \frac{\partial u_1}{\partial t} = d_1 \frac{\partial^2 u_1}{\partial x^2} + r_1 u_1(x, t) \cdot [1 - (a_1 + b_1) \cdot \\ u_1(x, t) - c_1 u_2(x, t)], \\ \frac{\partial u_2}{\partial t} = d_2 \frac{\partial^2 u_2}{\partial x^2} + r_2 u_2(x, t) \cdot [1 - (a_2 + b_2) \cdot \\ u_2(x, t) - c_2 u_1(x, t)] \end{cases} \quad (3)$$

方程 (3) 的行波解已有许多重要结果。特别是文献 [6-8] 等对连接  $E_0$  与  $E^*$  的行波解进行了研究, 这类行波解刻画了种群动力学中两个竞争物种的同步入侵。为了刻画一个入侵者和原住者之间的动力学性质, 需要研究连接  $E_1$  与  $E_2$  行波解的最小波速, 见文献 [9-12]。

当系统 (1) 仅仅涉及时间时滞, 则其形式之一如下

$$\begin{cases} \frac{\partial u_1}{\partial t} = d_1 \frac{\partial^2 u_1}{\partial x^2} + r_1 u_1(x, t) [1 - (a_1 + b_1) \cdot \\ u_1(x, t - \tau_1) - c_1 u_2(x, t - \tau_2)], \\ \frac{\partial u_2}{\partial t} = d_2 \frac{\partial^2 u_2}{\partial x^2} + r_2 u_2(x, t) [1 - (a_2 + b_2) \cdot \\ u_2(x, t - \tau_3) - c_2 u_1(x, t - \tau_4)] \end{cases} \quad (4)$$

这里  $\tau_1, \tau_2, \tau_3, \tau_4$  是非负常数。在文献 [13-15] 中, 证明了当  $\tau_1, \tau_3$  足够小时, 方程 (4) 行波解的存在性问题。文献 [16] 进一步考虑了涉及分布时滞的竞争扩散系统 (1) 的行波解。

近来, 文献 [13, 14] 中的方法被推广到方程 (1) 中。例如, 文献 [2] 研究了连接  $E_0$  与  $E^*$  的行波解的存在性。具体说来, 通过构造上下解, 文献 [2] 在如下条件下得到了行波解的存在性条件:

- (i) 波的速度  $c > c_0 := \max\{2\sqrt{d_1 r_1}, 2\sqrt{d_2 r_2}\}$ ;
- (ii) 当涉及种内竞争时, 时滞是非常的小同时非局部效应非常的微弱;
- (iii)  $g_1, g_3$  采取八种特殊形式。

事实上, 在许多模型研究中, 最小波速度是一种非常重要的工具<sup>[17-18]</sup>。也就是当波速小于某个阈值时, 方程 (1) 没有一个连接  $E_0$  与  $E^*$  的正的行波解, 但是该结论尚未被证明。同时对于一般的核函数  $g_i$ , 很难利用与文献 [2, 13-14] 中相似的方法去验证上下解。本文的目的就是研究方程 (1) 的最小波速度。

受文献 [16] 的启发, 通过构造容易被验证的广义上下解来证明行波解的存在性。为了研究行波解的渐近行为, 使用了渐近传播理论与压缩矩形的

思想。最后, 利用渐近传播理论证明了行波解的不存在性。因此在一定条件下证明了,  $c_0$  是连接  $E_0$  与  $E^*$  的行波解的最小波速, 这样就完善了文献 [2] 中的结果。

### 1 行波解的存在性

设  $X$  是如下函数空间

$X = \{u: u \text{ 是从 } \mathbf{R} \text{ 到 } \mathbf{R}^2 \text{ 有界一致连续函数}\}$

令  $\mu > 0$ ,  $\|\cdot\|$  表示  $\mathbf{R}^2$  中标准的上确界范数。定义

$$B_\mu(\mathbf{R}, \mathbf{R}^2) = \{u \in X: \sup_{\xi \in \mathbf{R}} \|u(\xi)\| e^{-\mu|\xi|} < \infty\}$$

并且  $|u|_\mu = \sup_{\xi \in \mathbf{R}} \|u(\xi)\| e^{-\mu|\xi|}$ 。则在定义范数  $|\cdot|_\mu$  后,  $B_\mu(\mathbf{R}, \mathbf{R}^2)$  是 Banach 空间。

这里所说的行波解是具有如下形式的特解

$$u_1(x, t) = \varphi(x + ct), u_2(x, t) = \psi(x + ct),$$

其中  $(\varphi, \psi) \in X$  是波廓,  $c > 0$  是波速。因此  $(\varphi, \psi)$  与  $c$  满足下列泛函微分系统

$$\begin{cases} c\varphi'(\xi) = d_1 \varphi''(\xi) + r_1 \varphi(\xi) [1 - a_1 \varphi(\xi) - \\ b_1 (g_1 * \varphi)(\xi) - c_1 (g_2 * \psi)(\xi)], \\ c\psi'(\xi) = d_2 \psi''(\xi) + r_2 \psi(\xi) [1 - a_2 \psi(\xi) - \\ b_2 (g_3 * \psi)(\xi) - c_2 (g_4 * \varphi)(\xi)] \end{cases} \quad (5)$$

其中

$$(g_i * \varphi)(\xi) = \int_0^\infty \int_{\mathbf{R}} g_i(y, s) \varphi(\xi - cs - y) dy ds,$$

$$i \in \{1, 2, 3, 4\}, \varphi \in \{\varphi, \psi\}$$

这样行波解问题就转化为方程 (5) 的非常数正解的存在性。对每个固定的  $c > c_0$ , 定义

$$\gamma_i = \frac{c - \sqrt{c^2 - 4d_i r_i}}{2d_i},$$

$$\gamma_{2+i} = \frac{c + \sqrt{c^2 - 4d_i r_i}}{2d_i}, i = 1, 2$$

并选择常数

$$\eta \in \left(1, \min\left\{\frac{\gamma_3}{\gamma_1}, \frac{\gamma_4}{\gamma_2}, \frac{\gamma_3 + \gamma_1}{\gamma_1}, \frac{\gamma_3 + \gamma_1}{\gamma_3}, \frac{c_0}{2d_1 \gamma_1}, \frac{c_0}{2d_2 \gamma_2}\right\}\right)$$

利用这些常数以及  $q > 1$ , 定义连续函数

$$\begin{aligned} \bar{\varphi}(\xi) &= \min\{e^{\gamma_1 \xi}, 1/a_1\}, \bar{\varphi}(\xi) = \max\{e^{\gamma_1 \xi} - qe^{\eta \gamma_1 \xi}, 0\}, \\ \bar{\psi}(\xi) &= \min\{e^{\gamma_2 \xi}, 1/a_2\}, \bar{\psi}(\xi) = \max\{e^{\gamma_2 \xi} - qe^{\eta \gamma_2 \xi}, 0\} \end{aligned}$$

为方便起见还记  $\xi_1 = \frac{-\ln a_1}{\gamma_1}, \xi_2 = \frac{-\ln q}{(\eta - 1)\gamma_1}, \xi_3 = \frac{-\ln a_2}{\gamma_2}, \xi_4 = \frac{-\ln q}{(\eta - 1)\gamma_2}$ 。

引理 1 若

$$\int_0^\infty \int_{\mathbf{R}} g_i(y, s) e^{\gamma(cs+y)} dy ds < \infty, \gamma \in \left(0, \frac{c_0}{2}\right) \quad (6)$$

$$q = 1 + \frac{a_1 + b_1 \int_0^\infty \int_{\mathbb{R}} g_1(y, s) e^{\gamma_1(y+cs)} dy ds}{c\eta\gamma_1 - d_1\eta^2\gamma_1^2 - r_1} + \frac{c_1 \int_0^\infty \int_{\mathbb{R}} g_2(y, s) e^{\gamma_2(y+cs)} dy ds}{c\eta\gamma_1 - d_1\eta^2\gamma_1^2 - r_1} + \frac{a_2 + b_2 \int_0^\infty \int_{\mathbb{R}} g_3(y, s) e^{\gamma_2(y+cs)} dy ds}{c\eta\gamma_1 - d_1\eta^2\gamma_2^2 - r_1} + \frac{c_2 \int_0^\infty \int_{\mathbb{R}} g_4(y, s) e^{\gamma_1(y+cs)} dy ds}{c\eta\gamma_2 - d_1\eta^2\gamma_2^2 - r_2} > 1$$

则

$$c\bar{\varphi}'(\xi) \geq d_1\bar{\varphi}''(\xi) + r_1\bar{\varphi}(\xi) [1 - a_1\bar{\varphi}(\xi) - b_1(g_1 * \bar{\varphi})(\xi) - c_1(g_2 * \bar{\psi})(\xi)], \xi \neq \xi_1 \quad (7)$$

$$c\bar{\varphi}'(\xi) \leq d_1\bar{\varphi}''(\xi) + r_1\bar{\varphi}(\xi) [1 - a_1\bar{\varphi}(\xi) - b_1(g_1 * \bar{\varphi})(\xi) - c_1(g_2 * \bar{\psi})(\xi)], \xi \neq \xi_2 \quad (8)$$

$$c\bar{\psi}'(\xi) \geq d_1\bar{\psi}''(\xi) + r_2\bar{\psi}(\xi) [1 - a_2\bar{\psi}(\xi) - b_2(g_3 * \bar{\psi})(\xi) - c_2(g_4 * \bar{\varphi})(\xi)], \xi \neq \xi_3 \quad (9)$$

$$c\bar{\psi}'(\xi) \leq d_1\bar{\psi}''(\xi) + r_2\bar{\psi}(\xi) [1 - a_2\bar{\psi}(\xi) - b_2(g_3 * \bar{\psi})(\xi) - c_2(g_4 * \bar{\varphi})(\xi)], \xi \neq \xi_4 \quad (10)$$

**证明** 先证 (7) 式。如果  $\xi > \xi_1$ , 则  $\bar{\varphi}(\xi) = 1/a_1$  并且

$$1 - a_1\bar{\varphi}(\xi) - b_1(g_1 * \bar{\varphi})(\xi) - c_1(g_2 * \bar{\psi})(\xi) \leq 0$$

因此 (7) 式显然成立。如果  $\xi < \xi_1$ , 则  $\bar{\varphi}(\xi) = e^{\gamma_1\xi}$  并且

$$c\bar{\varphi}'(\xi) = c\gamma_1 e^{\gamma_1\xi} = e^{\gamma_1\xi} [d_1\gamma_1^2 + r_1] = d_1\bar{\varphi}''(\xi) + r_1\bar{\varphi}(\xi)$$

由  $\gamma_1$  的定义则可完成 (7) 式的证明。同理可得 (9) 式。

如果  $\xi > \xi_2$ , 则  $\bar{\varphi}(\xi) = 0$  并且 (8) 式显然成立。否则,  $\xi < \xi_2 < 0$  满足

$$r_1\bar{\varphi}(\xi) [-a_1\bar{\varphi}(\xi) - b_1(g_1 * \bar{\varphi})(\xi) - c_1(g_2 * \bar{\psi})(\xi)] \geq r_1\bar{\varphi}(\xi) [-a_1 e^{\gamma_1\xi} - b_1(g_1 * e^{\gamma_1\cdot})(\xi) - c_1(g_2 * e^{\gamma_2\cdot})(\xi)] = r_1\bar{\varphi}(\xi) \cdot [-a_1 e^{\gamma_1\xi} - b_1 e^{\gamma_1\xi} \int_0^\infty \int_{\mathbb{R}} g_1(y, s) e^{\gamma_1(y+cs)} dy ds - c_1 e^{\gamma_2\xi} \int_0^\infty \int_{\mathbb{R}} g_2(y, s) e^{\gamma_2(y+cs)} dy ds] \geq r_1 e^{\gamma_1\xi} [-a_1 e^{\gamma_1\xi} - b_1 e^{\gamma_1\xi} \int_0^\infty \int_{\mathbb{R}} g_1(y, s) e^{\gamma_1(y+cs)} dy ds - c_1 e^{\gamma_2\xi} \int_0^\infty \int_{\mathbb{R}} g_2(y, s) e^{\gamma_2(y+cs)} dy ds] \geq r_1 e^{\gamma_1\xi} [-a_1 - b_1 \int_0^\infty \int_{\mathbb{R}} g_1(y, s) e^{\gamma_1(y+cs)} dy ds - c_1 \int_0^\infty \int_{\mathbb{R}} g_2(y, s) e^{\gamma_2(y+cs)} dy ds]$$

注意到

$$c\bar{\varphi}'(\xi) - d_1\bar{\varphi}''(\xi) - r_1\bar{\varphi}(\xi) = -q e^{\gamma_1\xi} [c\eta\gamma_1 - d_1\eta^2\gamma_1^2 - r_1]$$

则 (8) 式在  $q > 1$  满足如下条件时成立

$$-q [c\eta\gamma_1 - d_1\eta^2\gamma_1^2 - r_1] \leq -a_1 - b_1 \int_0^\infty \int_{\mathbb{R}} g_1(y, s) e^{\gamma_1(y+cs)} dy ds - c_1 \int_0^\infty \int_{\mathbb{R}} g_2(y, s) e^{\gamma_2(y+cs)} dy ds$$

同理可证 (10) 式。引理证毕。

定义集合

$$\Gamma = \{(\varphi, \psi) : (\varphi, \psi) \in X \text{ 和 } (\underline{\varphi}, \underline{\psi}) \leq (\varphi, \psi) \leq (\bar{\varphi}, \bar{\psi})\}$$

则  $\Gamma$  是凸的非空集合, 而且在  $|\cdot|_\mu$  范数意义下是有界闭集。

为方便起见, 对有界一致连续函数  $\varphi, \psi, \phi$ , 定义

$$\begin{cases} f_1(\varphi, \psi, \phi)(\xi) = r_1\varphi(\xi) [1 - a_1\varphi(\xi) - b_1(g_1 * \phi)(\xi) - c_1(g_2 * \psi)(\xi)], \\ f_2(\varphi, \psi, \phi)(\xi) = r_2\psi(\xi) [1 - a_2\psi(\xi) - b_2(g_3 * \phi)(\xi) - c_2(g_4 * \varphi)(\xi)] \end{cases}$$

则  $f_1$  关于  $\phi, \psi$  是一个单调减函数,  $f_2$  关于  $\varphi, \psi$  是一个单调减函数。

令  $\beta > 0$  使得

$$\begin{aligned} \beta u + r_1 u [1 - a_1 u - b_1/a_1 - c_1/a_2], \\ \beta v + r_2 v [1 - a_2 v - b_2/a_2 - c_2/a_1] \end{aligned}$$

关于  $u \in [0, 1/a_1], v \in [0, 1/a_2]$  均单调增。

定义常数

$$\begin{aligned} \lambda_1 &= \frac{c - \sqrt{c^2 + 4\beta d_1}}{2d_1}, \lambda_2 = \frac{c + \sqrt{c^2 + 4\beta d_1}}{2d_1}, \\ \lambda_3 &= \frac{c - \sqrt{c^2 + 4\beta d_2}}{2d_2}, \lambda_4 = \frac{c + \sqrt{c^2 + 4\beta d_2}}{2d_2} \end{aligned}$$

以及算子  $P = (P_1, P_2) : \Gamma \rightarrow X$  :

$$\begin{cases} P_1(\varphi, \psi)(\xi) = \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{-\infty}^{\xi} e^{\lambda_1(\xi-s)} + \int_{\xi}^{\infty} e^{\lambda_2(\xi-s)} \right] [\beta\varphi(s) + f_1(\varphi, \psi, \varphi)(s)] ds, \\ P_2(\varphi, \psi)(\xi) = \frac{1}{d_2(\lambda_4 - \lambda_3)} \left[ \int_{-\infty}^{\xi} e^{\lambda_3(\xi-s)} + \int_{\xi}^{\infty} e^{\lambda_4(\xi-s)} \right] [\beta\psi(s) + f_2(\varphi, \psi, \psi)(s)] ds \end{cases} \quad (11)$$

其中  $(\varphi, \psi) \in \Gamma$ 。显然  $P$  的一个不动点就是方程 (5) 的解。

**注 1** 积分算子  $P$  在行波解理论中的应用, 参见文献 [19-22]。

**引理 2** 如果  $\mu \leq \min\{-\lambda_1, -\lambda_3\}$ , 则  $P: \Gamma \rightarrow \Gamma$  在  $l \cdot l_\mu$  范数意义下是全连续的。

**证明** 先验证  $P: \Gamma \rightarrow \Gamma$ 。对任意  $(\varphi, \psi) \in \Gamma$ , 由  $f_1$  的单调性与 (7) 式可知

$$P_1(\varphi, \psi)(\xi) = \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{\mathbf{R}} \min\{e^{\lambda_1(\xi-s)}, e^{\lambda_2(\xi-s)}\} [\beta\varphi(s) + f_1(\varphi, \psi, \varphi)(s)] ds \geq \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{\mathbf{R}} \min\{e^{\lambda_1(\xi-s)}, e^{\lambda_2(\xi-s)}\} [\beta\varphi(s) + f_1(\varphi, \bar{\psi}, \bar{\varphi})(s)] ds \geq \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{s \neq \xi_2} \min\{e^{\lambda_1(\xi-s)}, e^{\lambda_2(\xi-s)}\} [\beta\varphi(s) + c\varphi'(s) - d_1\varphi''(s)] ds = \varphi(\xi) - \frac{\min\{e^{\lambda_1(\xi-\xi_2)}, e^{\lambda_2(\xi-\xi_2)}\}}{(\lambda_2 - \lambda_1)} \varphi'(\xi_2^-) > \varphi(\xi), \xi \in \mathbf{R}$$

同时, 由 (8) 式与  $f$  的单调性可以得到

$$P_1(\varphi, \psi)(\xi) = \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{\mathbf{R}} \min\{e^{\lambda_1(\xi-s)}, e^{\lambda_2(\xi-s)}\} [\beta\varphi(s) + f_1(\varphi, \psi, \varphi)(s)] ds \leq \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{\mathbf{R}} \min\{e^{\lambda_1(\xi-s)}, e^{\lambda_2(\xi-s)}\} [\beta\bar{\varphi}(s) + f_1(\bar{\varphi}, \bar{\psi}, \bar{\varphi})(s)] ds \leq \frac{1}{d_1(\lambda_2 - \lambda_1)} \left[ \int_{s \neq \xi_1} \min\{e^{\lambda_1(\xi-s)}, e^{\lambda_2(\xi-s)}\} [\beta\bar{\varphi}(s) + c\bar{\varphi}'(s) - d_1\bar{\varphi}''(s)] ds = \bar{\varphi}(\xi) - \frac{\gamma_1 \min\{e^{\lambda_1(\xi-\xi_1)}, e^{\lambda_2(\xi-\xi_1)}\}}{a_1(\lambda_2 - \lambda_1)} < \bar{\varphi}(\xi), \xi \in \mathbf{R}$$

类似可得

$$\underline{\psi}(\xi) \leq P_2(\varphi, \psi)(\xi) \leq \bar{\psi}(\xi), \xi \in \mathbf{R}$$

这就得到了  $P: \Gamma \rightarrow \Gamma$ 。

此外, 当  $\mu \leq \min\{-\lambda_1, -\lambda_3\}$  时, 该映射是全连续的<sup>[16, 19-20, 23]</sup>。引理证毕。

**定理 1** 假设  $c > c_0$  且 (6) 式成立, 则方程 (5) 有一个正解  $(\varphi, \psi)$  满足

$$0 < \varphi(\xi) < \frac{1}{a_1}, 0 < \psi(\xi) < \frac{1}{a_2}, \xi \in \mathbf{R}$$

且

$$\lim_{\xi \rightarrow -\infty} \varphi(\xi) e^{-\gamma_1 \xi} = \lim_{\xi \rightarrow -\infty} \varphi(\xi) e^{-\gamma_2 \xi} = 1 \quad (12)$$

**证明** 由 Schauder 不动点理论, 存在  $(\varphi, \psi)$  满足 (12) 式且有

$$0 \leq \varphi(\xi) \leq \frac{1}{a_1}, 0 \leq \psi(\xi) \leq \frac{1}{a_2}, \xi \in \mathbf{R}$$

从引理 1 的证明可得

$$0 \leq \varphi(\xi) < \bar{\varphi}(\xi) = P_1(\varphi(\xi), \psi(\xi)) < \bar{\varphi}(\xi) \leq \frac{1}{a_1}$$

类似地, 当  $\xi \in \mathbf{R}$  时有  $0 < \psi(\xi) < 1/a_2$ 。证明结束。

## 2 行波解的渐近性态

现在利用文献 [16] 中的思想来研究非常数行波解的渐近性态。对于 Fisher 方程

$$\begin{cases} z_t(x, t) = Dz_{xx}(x, t) + Rz(x, t) \cdot [1 - z(x, t)/K], x \in \mathbf{R}, t > 0, \\ z(x, 0) = z(x), x \in \mathbf{R} \end{cases} \quad (13)$$

其中  $D, R, K$  均为正常数且  $z(x) > 0$  是一个具有非空支集的有界一致连续函数。

**引理 3**<sup>[24]</sup> 假定  $z(x, t)$  按方程 (13) 定义, 则对任意给定的  $\varepsilon \in (0, 2\sqrt{DR})$

$$\lim_{t \rightarrow \infty} \inf_{|x| < (2\sqrt{DR}-\varepsilon)t} z(x, t) = \lim_{t \rightarrow \infty} \sup_{|x| < (2\sqrt{DR}-\varepsilon)t} z(x, t) = K$$

本节假定

$$\frac{b_1}{a_1} + \frac{c_1}{a_2} < 1, \frac{b_2}{a_2} + \frac{c_2}{a_1} < 1$$

对  $s \in [0, 1], \varepsilon > 0$ , 定义

$y_i(s) = sk_i, z_i(s) = sk_i + (1-s)(1/a_i + \varepsilon), i=1, 2$  由文献 [25] 的引理 5.7.4 可知, 存在  $\varepsilon_0 > 0$  对任意的  $\varepsilon \in (0, \varepsilon_0)$ , 有

(C1)  $1 - a_i y_i(s) - b_i z_i(s) - c_i z_{3-i}(s) > 0, s \in (0, 1), i = 1, 2;$

(C2)  $1 - a_i z_i(s) - b_i y_i(s) - c_i y_{3-i}(s) < 0, s \in (0, 1), i = 1, 2。$

**引理 4** 假定  $(\varphi, \psi)$  是方程 (5) 的正解, 则

$$\liminf_{\xi \rightarrow \infty} \varphi(\xi) > 0, \liminf_{\xi \rightarrow \infty} \psi(\xi) > 0$$

**证明** 由定理 1 可以看出

$$c\varphi'(\xi) \geq d_1\varphi''(\xi) + r_1\varphi(\xi) \cdot [1 - b_1/a_1 - c_1/a_2 - a_1\varphi(\xi)]$$

由行波解的定义,  $u_1(x, t)$  满足

$$\begin{cases} \frac{\partial u_1}{\partial t} \geq d_1 \frac{\partial^2 u_1}{\partial x^2} + r_1 u_1(x, t) [1 - b_1/a_1 - c_1/a_2 - a_1 u_1(x, t)], x \in \mathbf{R}, t > 0, \\ u_1(x, 0) = \varphi(x) > 0, x \in \mathbf{R} \end{cases} \quad (14)$$

由引理 3 与比较原理<sup>[26]</sup>, 可得

$$\liminf_{t \rightarrow \infty} u_1(0, t) \geq [1 - b_1/a_1 - c_1/a_2]/a_1 > 0$$

从行波解的定义与  $u_1(0, t) = \varphi(ct)$  可以得到

$$\liminf_{\xi \rightarrow \infty} \varphi(\xi) \geq [1 - b_1/a_1 - c_1/a_2]/a_1 > 0$$

类似可得  $\liminf_{\xi \rightarrow \infty} \psi(\xi) \geq [1 - b_2/a_2 - c_2/a_1]/a_2 > 0$ 。引理证毕。

**定理 2** 假定  $(\varphi, \psi)$  如定理 1 中定义, 则

$$\lim_{\xi \rightarrow \infty} (\varphi(\xi), \psi(\xi)) = (k_1, k_2)。$$

**证明** 由引理 4 可以看出

$$(0,0) \ll \liminf_{\xi \rightarrow \infty} (\varphi(\xi), \psi(\xi)) \leq \limsup_{\xi \rightarrow \infty} (\varphi(\xi), \psi(\xi)) \leq (1/a_1, 1/a_2)$$

因此, 存在  $s_0 \in (0, 1]$  满足

$$(y_1(s_0), y_2(s_0)) \leq \liminf_{\xi \rightarrow \infty} (\varphi(\xi), \psi(\xi)) \leq \limsup_{\xi \rightarrow \infty} (\varphi(\xi), \psi(\xi)) \leq (z_1(s_0), z_2(s_0))$$

令  $s_1 = \max\{s_0\}$ , 则有  $s_1 \in (0, 1]$ 。现在证明  $s_1 = 1$ 。如果  $s_1 \in (0, 1)$ , 不失一般性, 可以假设  $\liminf_{\xi \rightarrow \infty} \varphi(\xi) = y_1(s_1)$ , 因此存在  $\{\xi_m\}, m \in \mathbf{N}$ , 且  $\lim_{m \rightarrow \infty} \xi_m = \infty$ , 满足

$$\liminf_{\xi \rightarrow \infty} \varphi(\xi) = \lim_{m \rightarrow \infty} \varphi(\xi_m) = y_1(s_1)$$

与

$$\liminf_{m \rightarrow \infty} (d_1 \varphi''(\xi_m) - c \varphi'(\xi_m)) \geq 0$$

同时, (C1) 表明

$$\lim_{m \rightarrow \infty} [1 - a_1 \varphi(\xi_m) - b_1 (g_1 * \varphi)(\xi_m) - c_1 (g_2 * \psi)(\xi_m)] \geq$$

$$1 - a_1 y_1(s_1) - b z_1(s_1) - c z_2(s_1) > 0$$

并且有

$$\lim_{m \rightarrow \infty} [d_1 \varphi''(\xi_m) - c \varphi'(\xi_m) + r_1 \varphi(\xi_m) \cdot [1 - a_1 \varphi(\xi_m) - b_1 (g_1 * \varphi)(\xi_m) - c_1 (g_2 * \psi)(\xi_m)]] > 0$$

矛盾, 故原结论成立。定理证毕。

### 3 行波解的不存在性

**定理 3** 对任意的  $c < c_0$ , 方程 (5) 没有有界正解满足

$$\lim_{\xi \rightarrow \infty} \varphi(\xi) = \lim_{\xi \rightarrow \infty} \psi(\xi) = 0,$$

$$\liminf_{\xi \rightarrow \infty} \varphi(\xi) > 0, \liminf_{\xi \rightarrow \infty} \psi(\xi) > 0 \quad (15)$$

**证明** 如若不然, 则存在  $c_1 < c_0$ , 方程 (5) 有解  $(\varphi(\xi), \psi(\xi))$  满足 (15) 式。 $(\varphi(\xi), \psi(\xi))$  也是  $P$  的一个不动点, 则  $\varphi(\xi)$  与  $\psi(\xi)$  均为一致连续函数。与定理 1 的证明类似可得

$$0 < \varphi(\xi) < 1/a_1, 0 < \psi(\xi) < 1/a_2, \xi \in \mathbf{R}$$

并且有

$$0 < (g_1 * \varphi)(\xi) < 1/a_1,$$

$$0 < (g_2 * \psi(\xi)) < 1/a_2, \xi \in \mathbf{R}$$

不失一般性, 假定  $c_0 = 2 \sqrt{d_1 r_1}$ 。令  $2\varepsilon > 0$  且满足  $c_1^2 = 4d_1 r_1 (1 - 2\varepsilon)$ 。由  $\lim_{\xi \rightarrow \infty} \varphi(\xi) = \lim_{\xi \rightarrow \infty} \psi(\xi) = 0$  可知, 存在  $T > 0$  使得

$$b_1 (g_1 * \varphi)(\xi) + c_1 (g_2 * \psi)(\xi) < \varepsilon, \xi \leq T$$

同时, 由于  $\varphi(\xi)$  是正的且  $\liminf_{\xi \rightarrow \infty} \varphi(\xi) > 0$ , 所以存在  $\varepsilon_0 > 0$  满足  $\varphi(\xi) > \varepsilon_0, \xi \geq T$ 。

因此有

$$b_1 (g_1 * \varphi)(\xi) + c_1 (g_2 * \psi)(\xi) < b_1/a_1 + c_1/a_2 = (b_1/a_1 + c_1/a_2) \varepsilon_0/\varepsilon_0 \leq (b_1/a_1 + c_1/a_2) \varphi(\xi)/\varepsilon_0, \xi \geq T$$

综上所述,  $\varphi(\xi)$  满足

$$d_1 \varphi''(\xi) - c_1 \varphi'(\xi) + r_1 \varphi(\xi) [1 - \varepsilon - (a_1 + (b_1/a_1 + c_1/a_2)/\varepsilon_0) \varphi(\xi)] \leq 0, \xi \in \mathbf{R}$$

由行波解的定义可知,  $u_1(x, t)$  满足

$$\begin{cases} \frac{\partial u_1}{\partial t} \geq d_1 \frac{\partial^2 u_1}{\partial x^2} + r_1 u_1(x, t) [1 - \varepsilon - (a_1 + (b_1/a_1 + c_1/a_2)/\varepsilon_0) u_1(x, t)], x \in \mathbf{R}, t > 0, \\ u_1(x, 0) = \varphi(x) > 0, x \in \mathbf{R} \end{cases}$$

由引理 3 与比较原理, 可以得到

$$\liminf_{t \rightarrow \infty} \inf_{|x| < ct} u_1(x, t) \geq \frac{1 - \varepsilon}{a_1 + (b_1/a_1 + c_1/a_2)/\varepsilon_0}$$

对任意满足  $d_1 x^2 - cx + r_1 (1 - \varepsilon) > 0, x > 0$  的  $c > 0$  成立。

令  $c_2^2 = 4d_1 r_1 (1 - 3\varepsilon/2)$ 。则有  $c_2 > c_1$  且  $d_1 x^2 - c_2 x + r_1 (1 - \varepsilon) > 0, x > 0$ 。因此有

$$\liminf_{t \rightarrow \infty} u_1(-c_2 t, t) \geq \frac{1 - \varepsilon}{a_1 + (b_1/a_1 + c_1/a_2)/\varepsilon_0}$$

取  $-x = c_2 t$ , 则有

$$x + c_1 t = (c_1 - c_2) t \rightarrow -\infty (t \rightarrow \infty)$$

该结果表明

$$u_1(-c_2 t, t) = \varphi(x + c_1 t) = \varphi((c_1 - c_2)t) \rightarrow 0 (t \rightarrow \infty)$$

矛盾。定理证毕。

**注 2** 定理 3 给出了行波解最小波速。

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